Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Thursday 16 May 2019

Afternoon | Paper Reference **8FM0-22**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
22: Further Pure Mathematics 2
(Part of option A only)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P61863A

1/1/1/1/1

P 6 1 8 6 3 A 0 1 1 6

Pearson

5.1: Eigenvalues & Eigenvectors
5.3: The Cayley-Hamilton Theorem

1. Given that

$$\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 2 & 2 \end{pmatrix}$$

(a) find the characteristic equation for the matrix **A**, simplifying your answer.

(2)

(b) Hence find an expression for the matrix $\mathbf{A}^{-1}$ in the form $\lambda\mathbf{A} + \mu\mathbf{I}$, where λ and μ are constants to be found.

(3)

a. characteristic equation : $\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{pmatrix} 3 & 2 \\ 2 & 2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 3-\lambda & 2 \\ 2 & 2-\lambda \end{pmatrix}$$

$$\det\left[\begin{pmatrix} 3-\lambda & 2 \\ 2 & 2-\lambda \end{pmatrix}\right] = 0$$

$\det(M)$ where $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\det(M) = (a)(d) - (b)(c)$

$(3-\lambda)(2-\lambda) - (2)(2) = 0$

$(6 - 2\lambda - 3\lambda + \lambda^2) - 4 = 0$

$\lambda^2 - 5\lambda + 2 = 0$ //

b. Use Cayley-Hamilton theorem:

characteristic eq^n $\lambda^2 - 5\lambda + 2 = 0$ can be written as $A^2 - 5A + 2I = 0$

$A^2 - 5A + 2I = 0$

$A - 5I + 2A^{-1} = 0$ $\div A$

$A^{-1} = \frac{5I - A}{2} = \frac{5}{2}I - \frac{1}{2}A$ (rearrange for A^{-1})

$A^{-1} = -\frac{1}{2}A + \frac{5}{2}I$ //

$\lambda = -\frac{1}{2}$ $\mu = \frac{5}{2}$

2. (i) Determine all the possible integers a, where $a > 3$, such that

$$15 \equiv 3 \bmod a$$

(2)

(ii) Show that if p is prime, x is an integer and $x^2 \equiv 1 \bmod p$ then either

$$x \equiv 1 \bmod p \qquad \text{or} \qquad x \equiv -1 \bmod p$$

(3)

(iii) A company has £13 940 220 to share between 11 charities.

Without performing any division and showing all your working, decide if it is possible to share this money equally between the 11 charities.

(2)

i. $15 - 3 = ka$

$12 = ka \quad , k \in \mathbb{Z}$

a must be a divisor of 12 greater than 3.

$12: \cancel{1}, \cancel{2}, \cancel{3}, 4, 6, 12$

$a = 4, 6, 12$ //

b. $x^2 - 1$ is divisible by p.

$(x+1)(x-1)$ is divisible by p and since p is prime either $(x-1)$ is divisible by p or $(x+1)$ is divisible by p.

$\therefore x \equiv 1 \bmod p$ or $x \equiv -1 \bmod p$ //

must mention

iii. Must check if 13940220 is divisible by 11.

An integer is divisible by 11 if the alternating sum of its digits is 11.

$1 - 3 + 9 - 4 + 0 - 2 + 2 - 0 = 3$

$11 \nmid 3$ (3 is not divisible by 11)

$\therefore$ It is not possible to share the money equally between 11 charities //

3.1: Loci on an Argand Diagram
3.2: Regions in an Argand Diagram

3. A curve C in the complex plane is described by the equation

$$|z-1-8\mathrm{i}| = 3|z-1|$$

(a) Show that C is a circle, and find its centre and radius.

(4)

(b) Using the answer to part (a), determine whether $z = 3 - 3\mathrm{i}$ satisfies the inequality

$$|z-1-8\mathrm{i}| \geqslant 3|z-1|$$

(2)

(c) Shade, on an Argand diagram, the set of points that satisfies both

$$|z-1-8\mathrm{i}| \geqslant 3|z-1| \quad \text{and} \quad 0 \leqslant \arg(z+\mathrm{i}) \leqslant \frac{\pi}{4}$$

(4)

a. USE ALGEBRAIC APPROACH OF EVALUATING LOCI:

$|z-1-8i| = 3|z-1|$

let $z = x+iy$

$|x+iy-1-8i| = 3|x+iy-1|$

separate into real + imaginary parts

$|(x-1)+(y-8)i| = 3|(x-1)+(y)i|$

$\sqrt{(x-1)^2+(y-8)^2} = 3\sqrt{(x-1)^2+(y)^2}$

$\left(\sqrt{(x-1)^2+(y-8)^2}\right)^2 = \left(3\sqrt{(x-1)^2+(y)^2}\right)^2$

square both sides to get rid of sqrt $\sqrt{\ }$

$(x-1)^2+(y-8)^2 = (3)^2\left(\sqrt{(x-1)^2+(y)^2}\right)^2$

$(x-1)^2+(y-8)^2 = 9[(x-1)^2+(y)^2]$

$(x-1)^2+(y-8)^2 = 9(x-1)^2+9y^2$

$0 = 8(x-1)^2+9y^2-(y-8)^2$

$0 = 8(x^2-2x+1)+9y^2-(y^2-16y+64)$

$0 = 8x^2-16x+8+9y^2-y^2+16y-64$

$0 = 8x^2-16x+8y^2+16y-56$

$\div 8$

$0 = x^2-2x+y^2+2y-7$

$0 = (x-1)^2-1+(y+1)^2-1-7$

$0 = (x-1)^2+(y+1)^2-9$

$(x-1)^2+(y+1)^2 = 9$

rearrange to get circle eq^n

centre : $(1,-1)$

radius : $\sqrt{9} = 3$ units

Question 3 continued

b. $|z-1-8i| \geq 3|z-1|$

$z = 3-3i$

$|3-3i-1-8i| \geq 3|3-3i-1|$

$|2-11i| \geq 3|2-3i|$

Take modulus of both sides now and check if true.

LHS: $\sqrt{(2)^2+(-11)^2} = 5\sqrt{5}$

RHS: $3\sqrt{(2)^2+(-3)^2} = 3\sqrt{13}$

$5\sqrt{5} > 3\sqrt{13}$

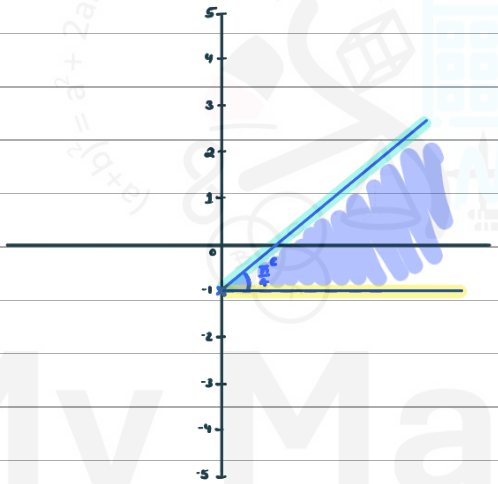
$\therefore$ $z = 3-3i$ satisfies the inequality $|z-1-8i| \geq 3|z-1|$

c. REMEMBER: IF $\geq$ OR $\leq$ USED, MUST USE STRAIGHT LINES.

IF $>$ OR $<$ USED, MUST USE DASHED LINES

$0 \leq \arg(z+i) \leq \frac{\pi}{4} \longrightarrow \{0 \leq \arg(z+i)\} \cap \{\arg(z+i) \leq \frac{\pi}{4}\}$

$\{0 \leq \arg(z-(-i))\} \cap \{\arg(z-(-i)) \leq \frac{\pi}{4}\}$



- plot the point $(0,-i)$
- draw horizontal dotted line from $(0,-i)$
- draw a line @ $\theta = 0^c$
- shade region above 0^c

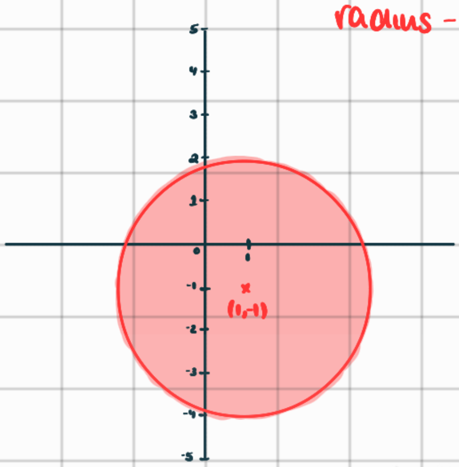
- plot the point $(0,-i)$
- draw horizontal dotted line from $(0,-i)$
- draw a line @ $\theta = \frac{\pi}{4}$
- shade region below $\frac{\pi}{4}$

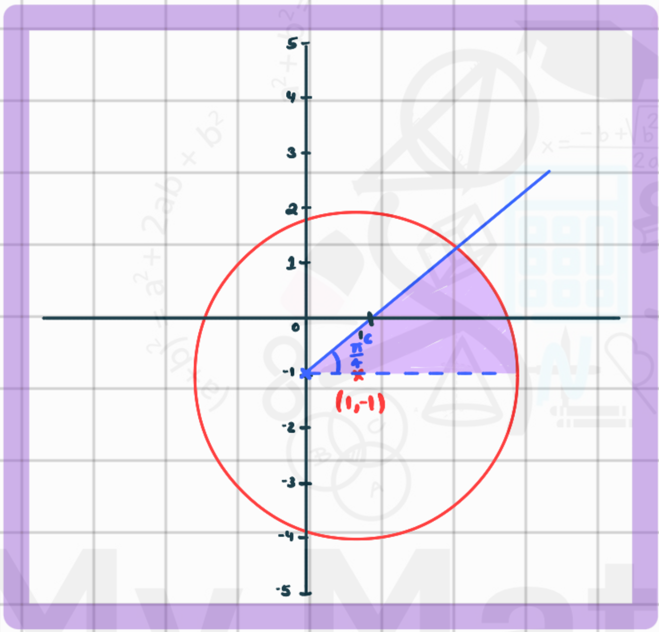
$|z-1-8i| \geq 3|z-1|$

shade region containing $(1,0)$

from part (a): centre - $(1,-1)$

radius - 3

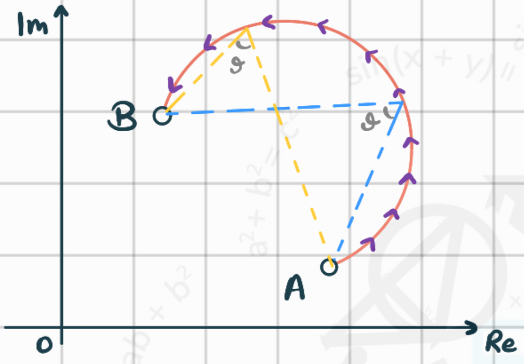




The locus of points z that satisfy $\arg\left(\frac{z-a}{z-b}\right) = \theta$

where $\theta \in \mathbb{R}$, $\theta > 0$ and $a, b \in \mathbb{C}$, is an arc of a circle with endpoints A and B representing the complex no.s a and b, respectively.

The locus is the arc of a circle drawn anticlockwise from A to B.



$$\arg\left(\frac{z-a}{z-b}\right) = \theta$$

if $\theta < \frac{\pi}{2}$ then the locus is a major arc of a circle

if $\theta = \frac{\pi}{2}$ then the locus is a semi-circle.

if $\theta > \frac{\pi}{2}$ then the locus is a minor arc of a circle

2.2: Cayley Tables & Finite Groups
2.3: Order & Subgroups

4. The set $\{e, p, q, r, s\}$ forms a group, A, under the operation $*$

Given that e is the identity element and that

$p*p = s$ $\quad$ $s*s = r$ $\quad$ $p*p*p = q$

(a) show that

(i) $p*q = r$

(ii) $s*p = q$

(2)

(b) Hence complete the Cayley table below.

*	e	p	q	r	s
e	e	p	q	r	s
p	p	s	r	e	q
q	q	r	p	s	e
r	r	e	s	q	p
s	s	q	e	p	r

A spare table can be found on page 11 if you need to rewrite your Cayley table.

(2)

(c) Use your table to find $p*q*r*s$

(1)

A student states that there is a subgroup of A of order 3

(d) Comment on the validity of this statement, giving a reason for your answer.

(2)

a. $p*q = p*p*p*p$

$= (p*p)*(p*p)$

$= s*s$

$= r$ // (shown)

$\therefore p*q = r$

$s*p = p*p*p$

$= p*p*p$

$= q$ // (shown)

$\therefore s*p = q$

P 6 1 8 6 3 A 0 1 0 1 6

Question 4 continued

c. $p * q * r * s$

$(p * q) * (r * s)$

$r * p$

$= e$ //

d. The order of the group is 5, and 3 does not divide by 5, so by Lagrange's theorem there can be no subgroup A of order 3. Therefore the student is wrong.

Only use this grid if you need to rewrite the Cayley table.

*	*e*	*p*	*q*	*r*	*s*
e					
p					
q					
r					
s					

(Total for Question 4 is 7 marks)

4.1: Forming Recurrence Relations 4.2: Solving First-Order Recurrence Relations

5. On Jim's 11th birthday his parents invest £1000 for him in a savings account.

The account earns 2% interest each year.

On each subsequent birthday, Jim's parents add another £500 to this savings account.

Let U_n be the amount of money that Jim has in his savings account n years after his 11th birthday, once the interest for the previous year has been paid and the £500 has been added.

(a) Explain, in the context of the problem, why the amount of money that Jim has in his savings account can be modelled by the recurrence relation of the form

$$U_n = 1.02U_{n-1} + 500 \qquad U_0 = 1000 \qquad n \in \mathbb{Z}^+$$

(3)

(b) State an assumption that must be made for this model to be valid.

(1)

(c) Solve the recurrence relation

$$U_n = 1.02U_{n-1} + 500 \qquad U_0 = 1000 \qquad n \in \mathbb{Z}^+$$

(5)

Jim hopes to be able to buy a car on his 18th birthday.

(d) Use the answer to part (c) to find out whether Jim will have enough money in his savings account to buy a car that costs £4 500

(2)

a. U_{n-1} is the amount in the saving account $n-1$ years after Jim's 11th birthday.
This is increased by 2% each year, so is $1.02U_{n-1}$.
Jim's parents invest £500 for each subsequent birthday so 500 is added.
$U_0 = 1000$ as this is the amount invested on Jim's birthday. //

b. Assumes that Jim does not withdraw any money from the savings account

c. $U_n = 1.02U_{n-1} + 500$

Homogenous part (C.F):
$U_n = 1.02U_{n-1}$
$U_n = c(1.02)^n$

Non-homogenous part (P.I):
$U_n = \lambda$
$U_{n-1} = \lambda$
} sub into U_n eq^n and solve for λ.

$$\lambda = 1.02\lambda + 500$$
$$0.02\lambda = -500$$
$$\lambda = -25000$$

Question 5 continued

gen sol^n: C.F. + P.I

gen sol^n: $u_n = c(1.02)^n - 25000$

u_0: $c(1.02)^0 - 25000 = 1000$ sub in $u_0 = 1000$

$c - 25000 = 1000$ to find value of c

$c = 26000$

Particular sol^n: $u_n = 26000(1.02)^n - 25000$

d. on his 11th birthday, it has been 7 years, $n = 7$

$u_7 = 26000(1.02)^7 - 25000$

$u_7 = 4865.827359$

$\approx$ £4865.83

$4865.83 > 4500$

$\therefore$ Jim will have enough money in his savings to buy a car costing £4500